UNIL | Université de Lausanne
Institut des sciences de la Terre

SCCER SoE
SWISS COMPETENCE CENTER for ENERGY RESEARCH
SUPPLY of ELECTRICITY

Università della Svizzera italiana
Institute of Computational Science ICS

Hydromechanical Coupling in Heterogeneous and Fractured Media

Task 3.2: Computational Energy Innovation

Marco Favino[1], Jürg Hunziker[1], Rolf Krause[2], Klaus Holliger[1]

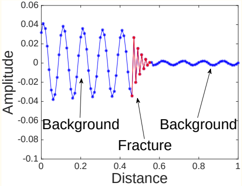
[1]Institute of Earth Sciences, University of Lausanne
[2]Institute of Computational Science, Università della Svizzera italiana

Lausanne, September 3-4, 2019

Fluid-saturated fractured porous rocks

Seismic wave propagation ⇒ Fluid pressure diffusion at mesoscale ⇒ Attenuation and velocity dispersion

Boundary conditions | Biot's equations | Output

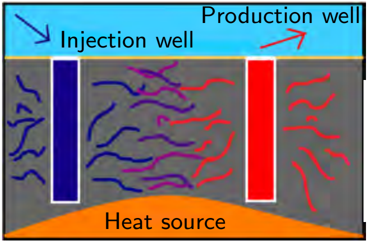


⇓

geometric + mechanical + hydraulic properties
of fracture networks in rock formations

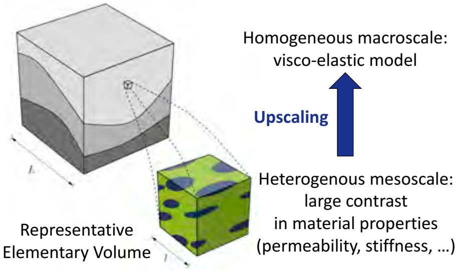
Applications

- geothermal energy production
- hydrocarbon exploration
- nuclear waste storage
- CO_2 sequestration



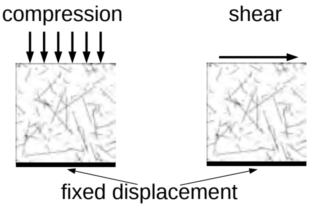
www.energybc.ca

Fractures as fluid pathways



Adapted from Jänicke et al. (2015)

Numerical Upscaling Experiments



periodic: displacement, stress, fluid pressure and the flux of the pore fluid

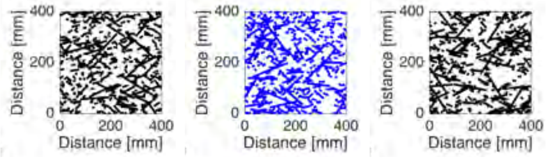
At the mesoscale (Rubino et al., 2011):

- perform time-harmonic oscillatory tests,
- for each frequency ω, compute attenuation and velocity dispersion Y_ω,
- use these obtained values to compute material properties for the macroscale problem.

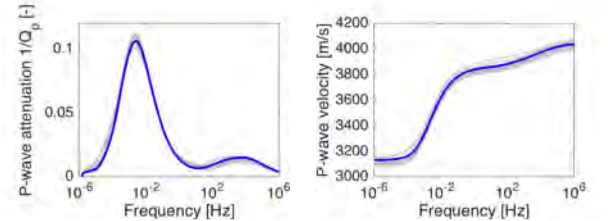
Fracture distribution in a Representative Elementary Volume

- deterministic information not available or insufficient ✗
- statistical properties ✓

Monte Carlo method: N samples to estimate $\mathbb{E}(Y_\omega)$



2D stochastic simulations in Hunziker, Favino et al., *J. Geophys. Res.* (2018)



Monte Carlo approximation

$$\frac{1}{N}\sum_{n=1}^{N} Y_\omega^n$$

Finite element discretization

Biot's equations

$$\left\{\begin{array}{r} -\nabla\cdot(\boldsymbol{\sigma}_E(\mathbf{u}) - \alpha p \mathbf{I}) = 0 \\ i\alpha\nabla\cdot\mathbf{u} + i\frac{p}{M} - \frac{1}{\omega}\nabla\cdot\left(\frac{k}{\eta}\nabla p\right) = 0 \end{array}\right.$$

Discretized Biot's equations

$$\left|\begin{array}{cc} \overbrace{A}^{\text{elasticity}} & \overbrace{-B^T}^{\textit{coupling}} \\ \underbrace{-iB}_{\textit{coupling}} & \underbrace{-iM - \frac{1}{\omega}C}_{\textit{diffusion}} \end{array}\right| \left|\begin{array}{c} \mathbf{u} \\ \mathbf{p} \end{array}\right| = \left|\begin{array}{c} \mathbf{f} \\ \mathbf{g} \end{array}\right|$$

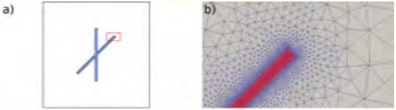
Root mean square error (RMSE)

$$\text{RMSE} \leq \underbrace{Ch^w}_{\text{Discretization error}} + \underbrace{DN^{-1/2}}_{\text{Statistical error}}, \qquad D^2 \approx Var(Y_\omega)$$

$\Rightarrow$ many expensive simulations on fine meshes

Meshing is one of the bottlenecks of the problem ✗

- elements follow the geometry
- hands-on
- time consuming
- may fail



Hierarchy of adapted meshes

Meshes

- do not have to resolve fractures
- can be “adapted” to any fracture distribution

Initial mesh (uniform)

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements

Hierarchy of adapted meshes

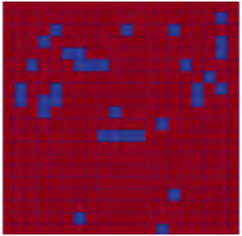
Initial mesh (uniform)

- Elements do not follow the geometry but refined close to the interfaces
- More elements where error is larger

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements

Hierarchy of adapted meshes



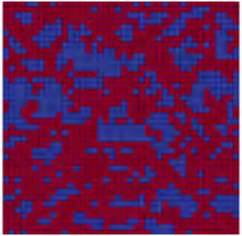
1 refinement step

- Elements do not follow the geometry but refined close to the interfaces
- More elements where error is larger

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements

Hierarchy of adapted meshes



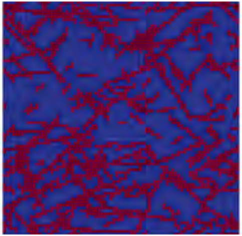
2 refinement steps

- Elements do not follow the geometry but refined close to the interfaces
- More elements where error is larger

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements

Hierarchy of adapted meshes



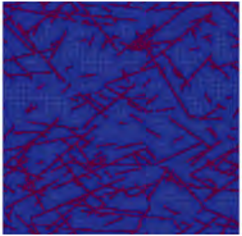
3 refinement steps

- Elements do not follow the geometry but refined close to the interfaces
- More elements where error is larger

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements

Hierarchy of adapted meshes



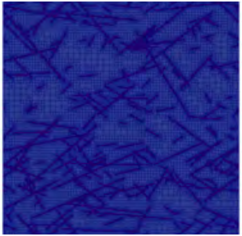
4 refinement steps

- Elements do not follow the geometry but refined close to the interfaces
- More elements where error is larger

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements

Hierarchy of adapted meshes



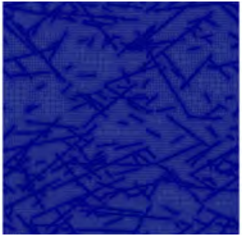
5 refinement steps

- Elements do not follow the geometry but refined close to the interfaces
- More elements where error is larger

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements

Hierarchy of adapted meshes



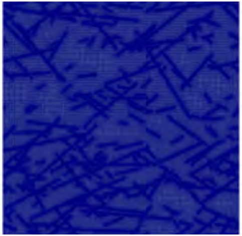
6 refinement steps

- Elements do not follow the geometry but refined close to the interfaces
- More elements where error is larger

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements

Hierarchy of adapted meshes

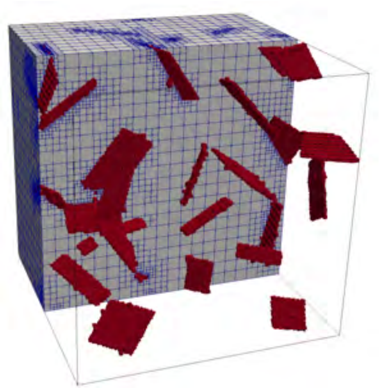


7 refinement steps

- Elements do not follow the geometry but refined close to the interfaces
- More elements where error is larger

Given a fracture distribution, we can apply an AMR algorithm:

1. select elements that have a non-emply overlap with at least one fracture
2. select neighbor elements such that the mesh is 1-irregular
3. refine selected elements



- Elements do not follow the interface between fractures and background
- Material properties may be discontinuous over some elements
- Properties are assigned per quadrature point at assembly time

$$C_{ij} = \int_{T_h} \frac{k}{\eta} \nabla \phi_j \cdot \nabla \phi_i \, dx = \sum_{qp} w_{qp} \frac{k(\mathbf{x}_{qp})}{\eta(\mathbf{x}_{qp})} \nabla \phi_j(\mathbf{x}_{qp}) \cdot \nabla \phi_i(\mathbf{x}_{qp})$$

- Reduced convergence rate (Babuška, 1970)

$$\|u - u_h\|_{H^1} \leq C h^{1/2}$$

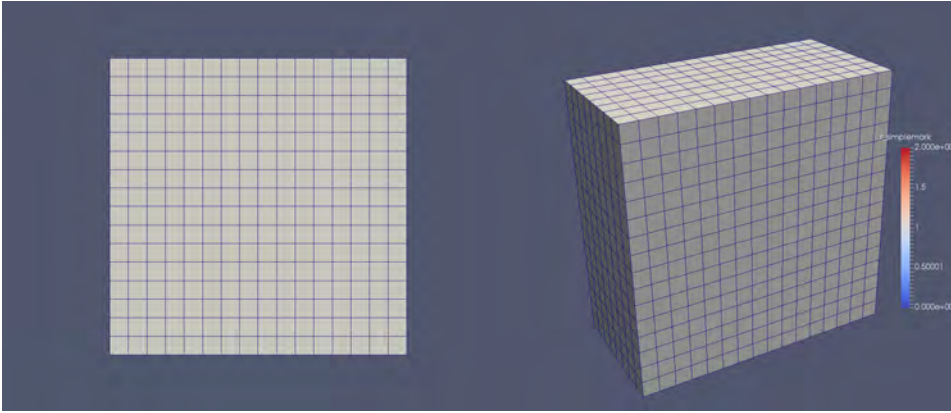
- Larger number of elements at the interfaces

Algorithm implemented in the FE framework MOOSE

- Developed a new app **Parrot**
- Extended MOOSE to work with complex-type variables
- AMR already available

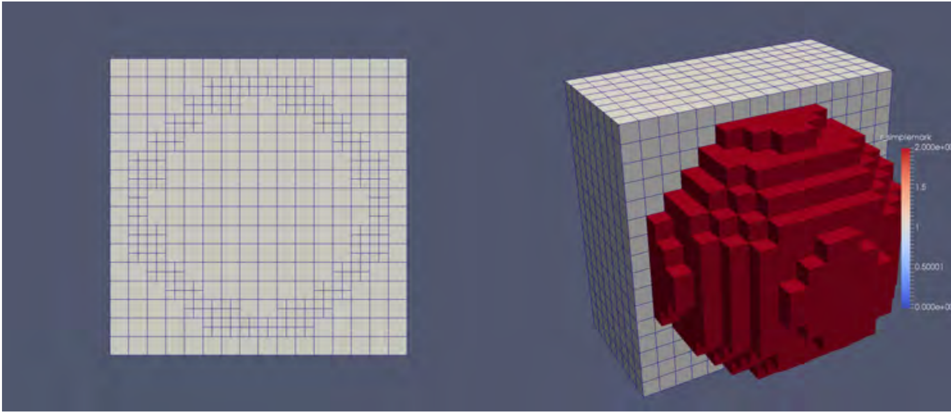
Algorithm validated in Favino et al. (2019, submitted)

- Horizontally layered medium (White et al., 1975)
- Spherically shaped gas inclusion in a cube (Pride et al., 2004)
- Two intersecting fractures
- Stochastic fracture networks

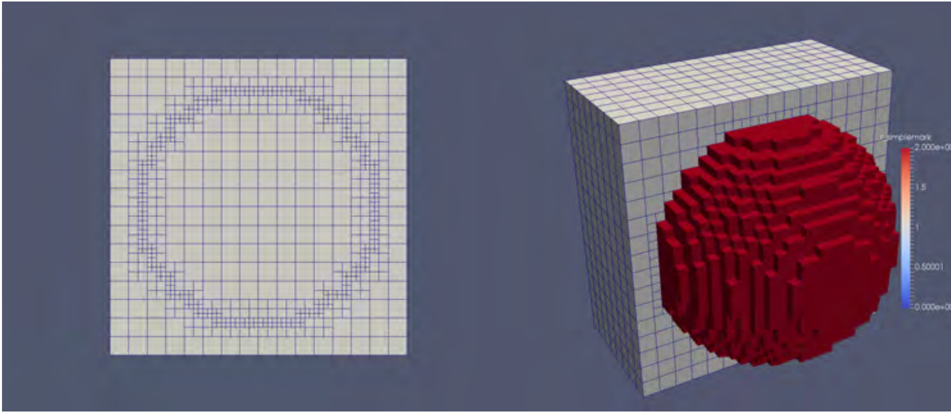


No. of nodes:	adaptive	uniform
	4913	4913

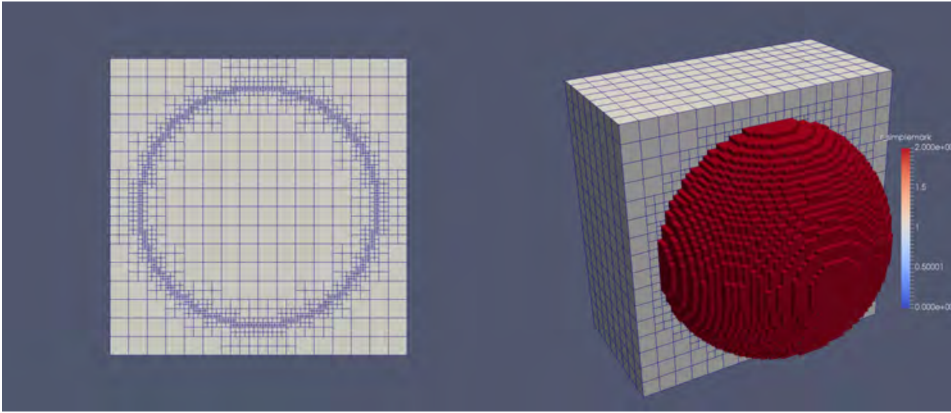
Spherically shaped gas inclusion



No. of nodes:	adaptive	uniform
	9528	35973

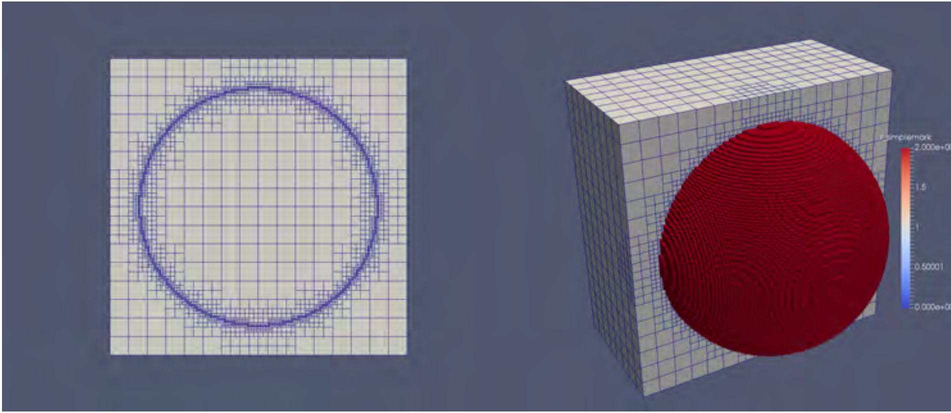


No. of nodes:	adaptive	uniform
	33944	274625

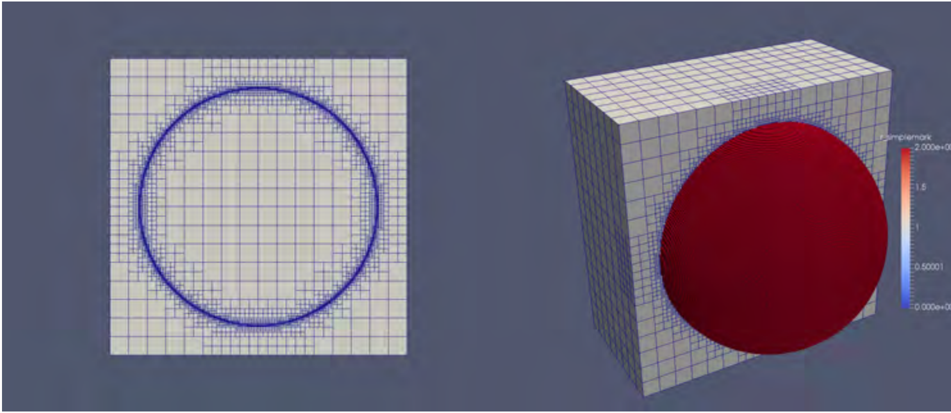


No. of nodes:	adaptive	uniform
	134464	2M

Unil | Université de Lausanne
Institut des sciences de la Terre

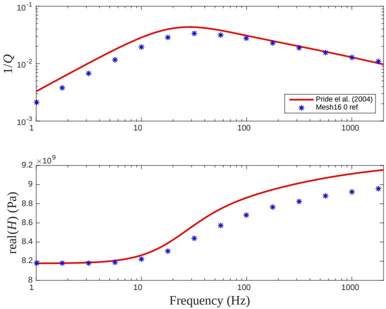


No. of nodes:	adaptive	uniform
	733279	16M



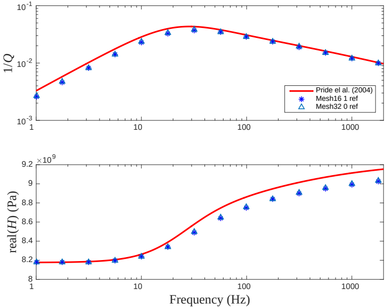
No. of nodes:	adaptive	uniform
	2.9M	135 M

Convergence to the analytical solution



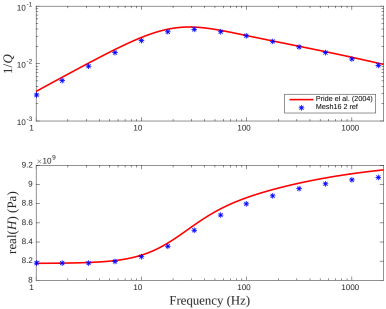
- reproduces the curves over the all spectrum
- no difference between uniform and adaptive refinement
- adaptive algorithm needed for
  - dispersion at small frequencies
  - attenuation at large frequencies

Convergence to the analytical solution



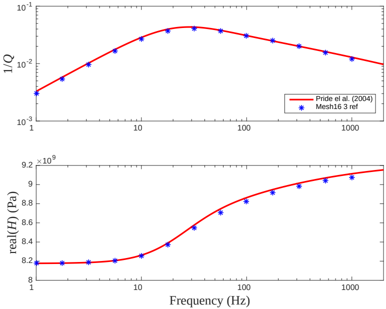
- reproduces the curves over the all spectrum
- no difference between uniform and adaptive refinement
- adaptive algorithm needed for
  - dispersion at small frequencies
  - attenuation at large frequencies

Convergence to the analytical solution



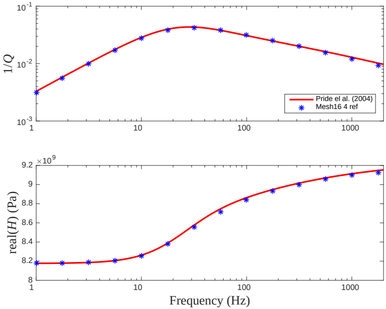
- reproduces the curves over the all spectrum
- no difference between uniform and adaptive refinement
- adaptive algorithm needed for
  - dispersion at small frequencies
  - attenuation at large frequencies

Convergence to the analytical solution

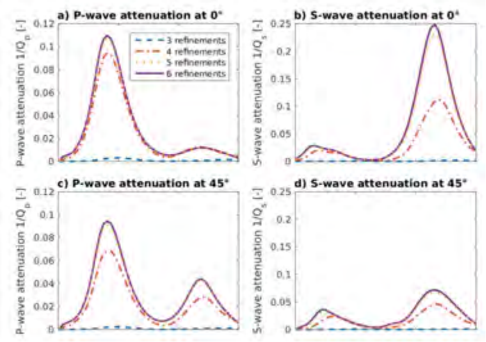


- reproduces the curves over the all spectrum
- no difference between uniform and adaptive refinement
- adaptive algorithm needed for
  - dispersion at small frequencies
  - attenuation at large frequencies

Convergence to the analytical solution

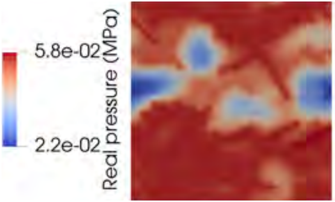


- reproduces the curves over the all spectrum
- no difference between uniform and adaptive refinement
- adaptive algorithm needed for
  - dispersion at small frequencies
  - attenuation at large frequencies

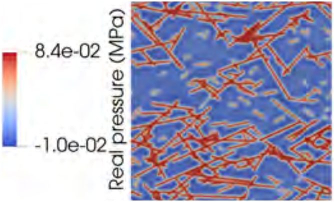


- Up to 3 mesh refinements no attenuation and velocity dispersion
- We cannot reproduce peaks related to background-to-fracture and fracture-to-fracture flows
- With 4 refinements, peaks are present but values are underestimated
- No difference between 5 and 6 mesh refinements

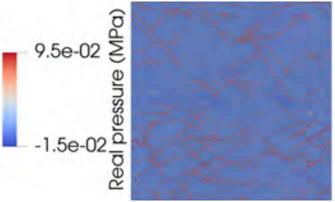
Simulated fluid pressure for different frequencies



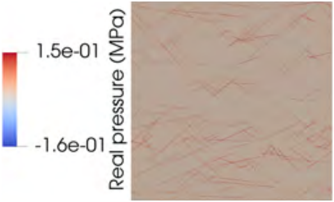
$\omega = 1e{-}4$ Hz



$\omega = 1e{-}2$ Hz



$\omega = 1e0$ Hz



$\omega = 1e4$ Hz

Conclusions

- Adaptive mesh refinement provides an automatized, foolproof mathod for meshing fractured media
- Software implementation in the FE framework MOOSE
- Already used in several follow-up studies, see e.g.
  - presentation Eva Caspari
  - poster Maria Nestola
  - poster Santiago Solazzi
  - poster Gabriel Quiroga

Future works

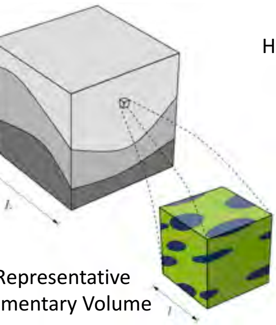
- Improve convergence of the discretization method, develop e.g, multiscale FE, composite FE, or partition of unity method
- A-posteriori error estimate for poroelasticity in time-frequency domain
- Develop an efficient solver exploiting the hierarchy of meshes created

Thank you for your attention

1. Development of a FE software to study
   - seismic attenuation
   - modulus dispersion

   due to fluid pressure diffusion in fractured rocks

2. Efficient for
   - stochastic fracture networks



pted from Jänicke et al. (2015)

Homogeneous macroscale:
visco-elastic model

Upscaling

Heterogenous mesoscale:
large contrast
in material properties
(permeability, stiffness, ...)

Biot's poroelasticity equations

$$\begin{cases} -\nabla \cdot (\boldsymbol{\sigma}_E(\mathbf{u}) - \alpha p \mathbf{I}) = 0 \\ i\alpha \nabla \cdot \mathbf{u} + i\frac{p}{M} - \frac{1}{\omega}\nabla \cdot \left(\frac{k}{\eta}\nabla p\right) = 0 \end{cases}$$

u: solid displacement
p: fluid pressure
ω: frequency

Time-harmonic oscillatory tests:

- attenuation
- velocity dispersion

$\Big\} Y_\omega$

Hybrid-dimensional model: 2D fractures

- simplified physics
- simple geometries

Biot's equations: 3D "thick" fractures

- complete coupled physics
- complex fracture geometries

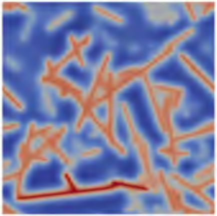
Model

- Biot's quasi-static equations
- fractured media (jumping parameters)
- time-frequency domain
  - **u** and p are complex variables

$$\begin{cases} -\nabla \cdot (2\mu\boldsymbol{\varepsilon} + \lambda \mathrm{tr}(\boldsymbol{\varepsilon})\mathbf{I} - \alpha p \mathbf{I}) & = & 0 \\ i\omega\alpha\nabla \cdot \mathbf{u} + i\omega\frac{p}{M} + \nabla \cdot \left(-\frac{k}{\eta}\nabla p\right) & = & 0 \end{cases}$$

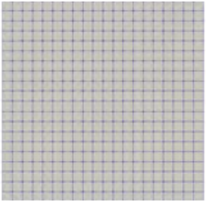
Computational challenges

- mesh generation
- efficient solution methods for complex FE
  - two different discretization approaches

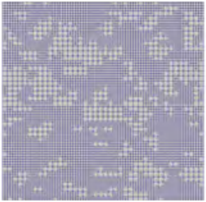


Adaptive mesh refinement for fracture networks

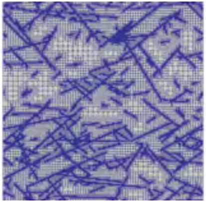
Hierarchy of adapted meshes



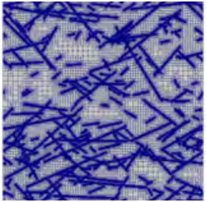
Initial mesh (uniform)



3 refinement steps



5 refinement steps



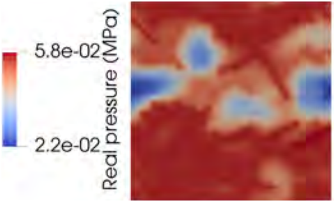
7 refinement steps

Algorithm implemented in MOOSE framework

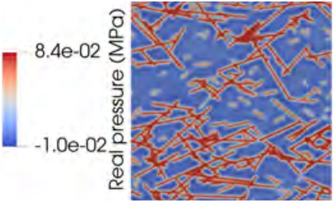
Validated in Favino et al., *J. Comput. Phys.* (2018, submitted)

UNIL | Université de Lausanne
Institut des sciences de la Terre

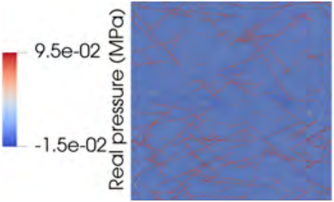
Simulated fluid pressure for different frequencies



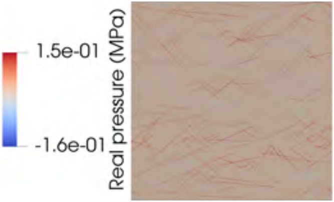
$\omega = 1\text{e}{-4}$ Hz



$\omega = 1\text{e}{-2}$ Hz



$\omega = 1\text{e}0$ Hz



$\omega = 1\text{e}4$ Hz

Algorithm implemented in MOOSE framework
Validated in Favino et al., *J. Comput. Phys.* (2018, submitted)

$$\begin{vmatrix} A & -B^T \\ -iB & -iM - \frac{1}{\omega}C \end{vmatrix} \begin{vmatrix} \mathbf{u} \\ \mathbf{p} \end{vmatrix} = \begin{vmatrix} \mathbf{f} \\ \mathbf{0} \end{vmatrix}$$

Complex FE

- 4 variables in 3D
- `complex<double>` type (two doubles for each entry)
- not well-conditioned
- better for factorization (direct solvers)
- Generalized Saddle-point problem
  - no energy
  - not symmetric $\Rightarrow$ no Lagrangian
  - requires ad-hoc solution methods
- e.g. Comsol

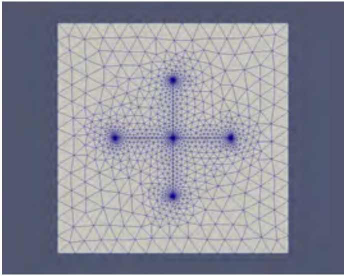
$$\begin{bmatrix} A & 0 & -B^T & 0 \\ 0 & A & 0 & -B^T \\ 0 & B & -\frac{1}{\omega}C & -M \\ B & 0 & M & -\frac{1}{\omega}C \end{bmatrix} \begin{bmatrix} \mathbf{u}_\mathrm{r} \\ \mathbf{u}_\mathrm{i} \\ \mathbf{p}_r \\ \mathbf{p}_i \end{bmatrix} = \begin{bmatrix} \mathbf{f}_\mathrm{r} \\ \mathbf{f}_\mathrm{i} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

Real FE

- 8 variables
- `double` type (one double per entry)
- better condition number
- better for iterative solvers
- Generalized Saddle-point problem
  - no energy
  - not symmetric $\Rightarrow$ no Lagrangian
  - requires ad-hoc solution methods

Multiscale problem:

- fracture thicknesses $\simeq 10^{-3}$ of domain size
- fractures need to be resolved to set correct parameters



- Meshing is one of the bottlenecks of the problem ✗
  - elements follow the geometry
  - hands-on
  - time consuming
  - may fail

$\Rightarrow$ unfeasible for realistic networks

UNIL | Université de Lausanne
Institut des sciences de la Terre

Mesh is generated once and then used for several frequencies

2D (5 parameters)

- center point (x,z)
- thickness and length
- dip around y-axis

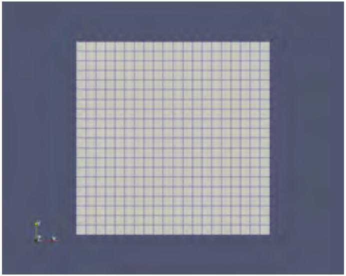
3D (8 parameters)

- center point (x,y,z)
- thickness, length and width
- dip around y-axis and x-axis

Parameters can be drawn from any distribution (e.g. de Dreuzy, Normal)

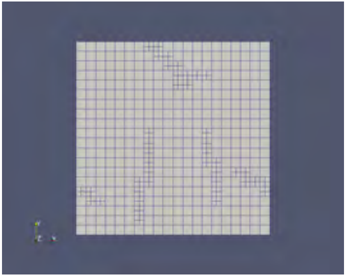
Example with 6 fractures

- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically



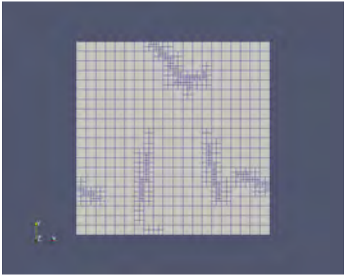
Example with 6 fractures

- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically



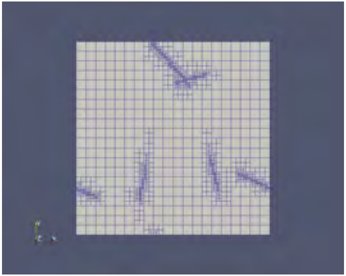
Example with 6 fractures

- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically



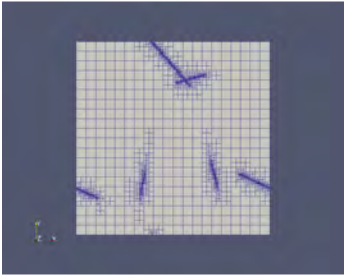
Example with 6 fractures

- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically



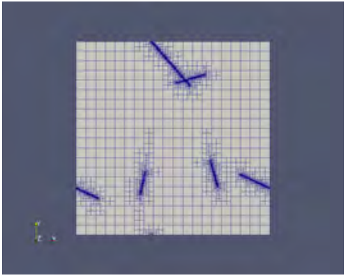
Example with 6 fractures

- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically



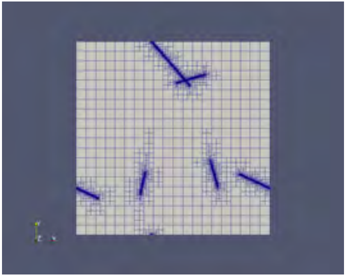
Example with 6 fractures

- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically



Example with 6 fractures

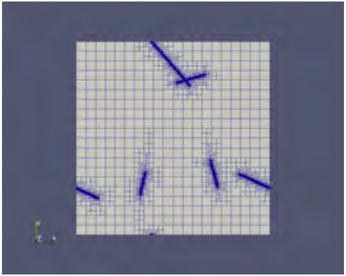
- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically

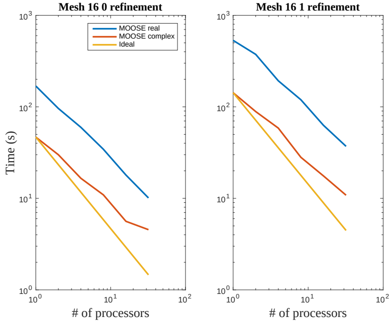


Unil | Université de Lausanne
Institut des sciences de la Terre

Example with 6 fractures

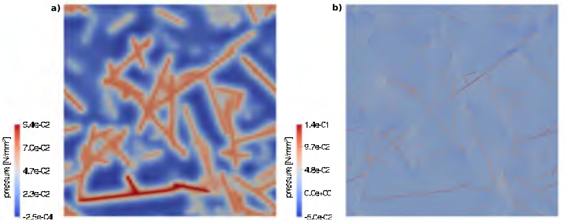
- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically



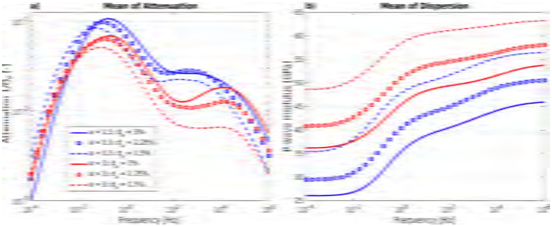


- better scaling for larger problems
- gain using complex MOOSE
    - from 2.2 to 3.6 for refinement 0
    - from 3.4 to 4.2 for refinement 1
- results with 4 refinements possible only with complex version

Unil | Université de Lausanne
Institut des sciences de la Terre



Real values of pressure and vertical real displacement at 10^{-1} and 10^{3} Hz



Parrot employed to compute

- displacement and pressure distributions
- dispersion and attenuation as functions of frequency
- mean value of 20 stochastic fracture networks
- see presentation by Eva Caspari and poster Jürg Hunziker